Guideline for the Adjustment and Evaluation of Survey Control

Special Publication 1

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Intergovernmental Committee on Surveying and Mapping (ICSM)

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Document History

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Table of contents

List of figures

List of tables

Terms and definitions

For the purpose of this Guideline, the terms and definitions listed below and those listed in the *Standard for the Australian Survey Control Network – Special Publication 1, Version 2.2* apply.

Term/Acronym	Definition
Confidence interval	A one dimensional range within which an estimated result is expected to fall at a particular level of confidence. It can also be thought of as the (±) range about the expected value which describes the level of uncertainty in a measurement or estimated result.
Confidence range	The elliptical or circular area describing the uncertainty in an estimated result at a particular level of confidence.
Coverage factor (k)	A statistical scalar used to modify a standard uncertainty to reach a greater (or expanded) level of confidence. The value assigned to k is chosen according to the desired level of confidence.
Critical value	Used in the context of testing reliability, it is the value at the extremity of a confidence interval or range. Estimated results exceeding this value are deemed to have failed the particular test of reliability.
Degrees of freedom (DoF)	A statistic that defines the redundancy of a least squares adjustment and equals the number of measurements minus the number of unknown parameters to be estimated.
Global test	An evaluation procedure performed on a least squares adjustment to assess the quality of the survey as a whole.
Local test	An evaluation procedure performed on individual survey measurements to assess the quality of a measurement and its assumed uncertainty.
Redundancy	A least squares adjustment is said to contain redundancy if the total number of measurements exceeds the minimum number required to compute the unknown parameters (i.e. when the degrees of freedom is greater than zero). When repeated measurements are taken to estimate an unknown parameter, the additional measurements are said to be redundant.
Sigma zero	The magnitude of the sum of the squares of the weighted measurement corrections arising from a least squares adjustment.
Uncertainty, expanded	Uncertainty expressed as a multiple of one standard deviation, multiplied by a coverage factor to produce a higher level of confidence.
Uncertainty, standard	Uncertainty expressed as one standard deviation.

1 About this Guideline

1.1 Introduction

The availability of accurate and reliable information relating to the position and uncertainty of Australia's survey control marks is critical to the integrity of the Australian Geospatial Reference System (AGRS). The purpose of this Guideline is to promote the adoption of uniform least squares adjustment and evaluation procedures to achieve the highest level of rigour and integrity in Australia's survey control mark network.

The generation of position and uncertainty for Australia's survey control marks relies upon a large number of measurements to these survey control marks using a variety of instruments, measurement techniques and processing software. Whilst a surveyor will always endeavour to obtain the most accurate and precise measurements to survey control marks, the true value of a survey control mark's position can never be measured nor derived with absolute certainty due to the inescapable presence of measurement error. In order to determine the single, most reliable position from a range of measurements whilst at the same time detecting and removing unacceptable measurement errors, the technique of least squares estimation should be used.

For geodetic control surveys, the least squares technique provides a reliable means for estimating the most probable value of survey control mark coordinates and uncertainties from a redundant set of measurements by satisfying predefined functional and stochastic models. The technique of least squares also provides a rigorous means for testing the estimated values and the measurements used to derive them.

This Guideline outlines ICSM's recommended procedures for using least squares to estimate and evaluate the position and uncertainty of survey control marks, to evaluate measurement uncertainty, and to evaluate network reliability. Examples are provided at the end of this Guideline.

1.2 Normative References

This Guideline should be read in conjunction with the *Standard for the Australian Survey Control Network – Special Publication 1, Version 2.2* herein referred to as the Standard.

The following documents may have relevance to the application of this Guideline.

International Guidelines

JCGM 100:2008, *Evaluation of Measurement Data – Guide to the Expression of Uncertainty in Measurement*, Joint Committee for Guides in Metrology – Bureau International des Poids et Mesures, Paris, France.

SP1 Standard

ICSM (2020), *Standard for the Australian Survey Control Network – Special Publication 1*, Version 2.2, Intergovernmental Committee on Surveying and Mapping, Canberra, Australia.

SP1 Guidelines

ICSM (2020), *Guideline for Control Surveys by Differential Levelling*, Version 2.2, Intergovernmental Committee on Surveying and Mapping, Canberra, Australia.

ICSM (2020), *Guideline for Control Surveys by GNSS*, Version 2.2, Intergovernmental Committee on Surveying and Mapping, Canberra, Australia.

ICSM (2020), *Guideline for Conventional Traverse Surveys*, Version 2.2, Intergovernmental Committee on Surveying and Mapping, Canberra, Australia.

ICSM Technical Manuals

ICSM (2020), *Geocentric Datum of Australia 2020 Technical Manual,* Intergovernmental Committee on Surveying and Mapping, Canberra, Australia.

ICSM (2007), *Australian Tides Manual – Special Publication 9,* Intergovernmental Committee on Surveying and Mapping, Wollongong, Australia.

2 Connection to datum

Survey control marks established for the AGRS shall be coordinated relative to the datums set out in Section 2 of the Standard.

2.1 Propagation of datum and uncertainty

In order to rigorously propagate datum and uncertainty throughout a survey control mark network, direct measurement to the AGRS survey control mark network is required.

For Datum Control Surveys, propagation of datum and uncertainty to new and existing survey control marks can be achieved by including the survey within AGRS adjustments (State, Territory and Australian Government).

For General Purpose Control Surveys, the jurisdictional agency responsible for maintaining the survey control mark network should make available, where possible, the relevant survey control information to facilitate the rigorous propagation of datum and uncertainty. Such information should include, at the least, datum aligned coordinates and heights, uncertainties and metadata for the survey control marks to which connection is being made.

Similarly, the jurisdictional agency responsible for maintaining the Australian Height Datum (AHD) should make available the relevant survey control information to facilitate the rigorous propagation of AHD. Such information should include AHD heights, quality estimates and metadata for the survey control marks to which a connection to AHD is being made.

The preferred means for propagating datum and uncertainty throughout an adjustment is to provide coordinates and a variance-covariance matrix which rigorously expresses the datum and quality (absolute and relative) of the existing survey control marks. However, in practice a full variance-covariance matrix is often unavailable. In such instances, *a priori* statistical information may be derived using empirical means or estimated values.

3 Expression of uncertainty

Section 4.2.1 of the Standard states that Survey Uncertainty (SU), Relative Uncertainty (RU) and Positional Uncertainty (PU) shall be expressed in terms of the 95% confidence level using any one of the following as appropriate:

- standard deviations;
- standard error ellipse/ellipsoid; or
- horizontal circular confidence region.

To express standard deviations for one dimensional components at the 95% confidence level, the uncertainty value is simply computed by scaling the estimated (1 sigma) standard deviation by coverage factor k = 1.960.

To express the (two dimensional) standard error ellipse at the 95% confidence level, the axes of the 95% error ellipse are obtained by scaling the (1 sigma) axes by coverage factor k = 2.448.

Similarly, the axes of the (three dimensional) 95% error ellipsoid are obtained by scaling the (1 sigma) axes by coverage factor k = 2.796.

For the horizontal circular confidence region, the 95% uncertainty value is calculated from the standard (1 sigma) error ellipse and is expressed as a single quantity, being the radius of the circular confidence region. The radius (r) of the circular confidence region is computed by:

$$r = a \times K$$

$$K = q_0 + q_1 C + q_2 C^2 + q_3 C^3$$

$$C = {}^{b}/_{a}$$

Where:

a = semi-major axis of the standard error ellipse

b = semi-minor axis of the standard error ellipse

q_0 = 1.960790

q_1 = 0.004071

q_2 = 0.114276

q_3 = 0.371625

Values for a and b shall be derived from the full *a-posteriori* variance matrix obtained from least squares adjustment.

4 Adjustment of survey control

4.1 Purpose

In the context of control surveys, the primary purpose of least squares adjustment is to estimate survey control mark coordinates from a set of measurements. In addition to this, least squares provides for the following additional outcomes:

- the uncertainty of the estimated coordinates;
- the quality of the survey control network; and
- the necessary statistics to evaluate the quality of the measurements and the estimated coordinates.

As defined in the Standard, the quality of estimated coordinates shall be expressed in terms of uncertainty. The following subsection outlines the recommended procedure for undertaking least squares adjustments in SU, PU and RU contexts.

4.2 Recommended procedure

Control surveys undertaken to derive the horizontal and vertical position of survey control marks relative to the AGRS should be adjusted in a rigorous least squares adjustment process. The following procedures are recommended:

(a) All survey control measurements should be corrected for all known calibration corrections and systematic error sources, and be accompanied by reliable values of uncertainty (or weights). To test the control survey for errors, redundant measurements sufficient to identify errors shall be used. The larger the degrees of freedom (DoF), the greater confidence can be gained from a survey.

(b) A minimally constrained adjustment should be tested using the local test (see Section 5.1) and global test (see Section 5.2).

(c) If required, estimated SU and associated RU values (or other reliable statistical methods) should be examined to assess whether the survey has achieved any predefined uncertainty or quality thresholds.

(d) When attempting to propagate datum and uncertainty, a fully constrained and appropriately weighted adjustment should be undertaken.

(e) The fully constrained adjustment should be tested using the local test (see Section 5.1), to verify that the imposed constraints do not result in measurement failure(s).

(f) The fully constrained adjustment should be tested using the global test (see Section 5.2). If this adjustment test fails, the quality of survey measurements

and constraints needs to be examined to identify and rectify the cause of failure.

(g) If required, estimated PU and associated RU values (or other reliable statistical methods) should be examined to assess whether the survey control network has achieved any predefined uncertainty or quality thresholds.

Whilst ICSM regards the circular form of PU as an acceptable means for simplifying the expression of *a-posteriori* uncertainty at the 95% confidence level, the circular form of uncertainty is not acceptable for use as *a-priori* statistical information to be used in constraining an adjustment in procedure (d).

5 Evaluating survey uncertainty

Statistical tests in a network adjustment are used to assess and/or demonstrate the reliability of a survey control network. The following two tests should be undertaken on both minimally constrained and fully constrained adjustments to evaluate the quality of a survey and the derived results.

5.1 Local test

A local test is an evaluation procedure performed on individual survey measurements to assess the quality of a measurement and its assumed (or *a-priori*) uncertainty. To validate each measurement and assumed uncertainty, the size of each adjusted measurement correction should be tested to verify that the correction lies within the upper and lower limits of the specified confidence interval. Such testing should be undertaken to demonstrate that:

- no gross errors exist within the measurements;
- all measurements are appropriately weighted;
- all measurements satisfy any predefined measurement precision criteria; and
- in the case of constrained adjustment, the imposed constraints are statistically reliable.

Local tests should be conducted using the Normal distribution at the 95% confidence level. A correction which exceeds the upper 95% confidence limit indicates a failure and the need to re-evaluate the assumed uncertainty of the measurements and/or imposed constraints.

5.2 Global (or sigma-zero) test

A global test is an evaluation procedure performed on a least squares adjustment to assess the quality of the survey as a whole. To validate the network as a whole, the sum of the squares of the weighted measurement corrections resulting from the adjustment (the adjustment result) should be tested against the DoF of the network. Testing should be conducted using the Chi-square distribution.

If the adjustment result is equal to the DoF, the sigma-zero (or variance factor) will be unity, indicating that the system of survey measurements, uncertainties and constraints is statistically reliable. Values larger than one indicate that one or more of the *a-priori* measurement uncertainties are over-optimistic, or a larger-than-expected correction has resulted. Values less than one suggest that the measurements were better than assumed by the combined set of measurement uncertainties. Values which exceed the upper confidence limit indicate a failure and the need to re-evaluate the uncertainty of the measurements (via the local test) and/or the uncertainty of the imposed constraints.

Whilst it is not always possible, nor essential, to achieve sigma-zero values equal to unity, a pass for both local and global tests is a minimum requirement for demonstrating survey control network reliability.

6 Example test procedure

To evaluate the quality of a control survey, the survey should be tested using SU, PU and RU as appropriate, in accordance with the survey measurement Guidelines relating to the desired level of quality.

ICSM recommends the following procedure to evaluate the coordinates and uncertainties of marks in a survey control network using least squares:

(a) Confirm that the adopted procedures for each measurement technique are commensurate with the desired quality outcome. Refer to the relevant survey Guidelines for guidance on selecting survey equipment and procedures appropriate for desired quality outcomes.

(b) Using the global test (see Section 5.2), confirm the sigma-zero value obtained from the minimally constrained adjustment is within the upper and lower limits of the 95% confidence interval.

(c) Confirm all measurements pass the local test at the 95% confidence level (see Section 5.1) in the minimally constrained adjustment.

(d) When propagating datum and uncertainty from the AGRS or including a survey within the AGRS, undertake a fully constrained adjustment using appropriate constraints, and:

  (i) Confirm the sigma-zero value obtained from the global test is within the upper and lower limits of the 95% confidence interval

  (ii) Using the local test, confirm the measurements still satisfy the expected measurement uncertainties at the 95% confidence level

(e) As required, evaluate the standard deviations, error ellipses, PU and RU at the 95% confidence level for all survey control marks.

6.1 Example: GNSS and terrestrial control survey

Figure 1 shows a small, high precision General Purpose Control Survey comprised of GNSS and terrestrial (horizontal angle, vertical angle, slope distance and differential levelling) measurements. The purpose of the survey is to establish four new marks from two AGRS marks – one survey control mark and one GNSS CORS site. The list of survey control marks and coordinates is shown in Table 1, and the full set of measurements is shown in Table 2 and Table 3. Note that this example is intended to demonstrate a rigorous adjustment and does not promote or infer optimal network design.

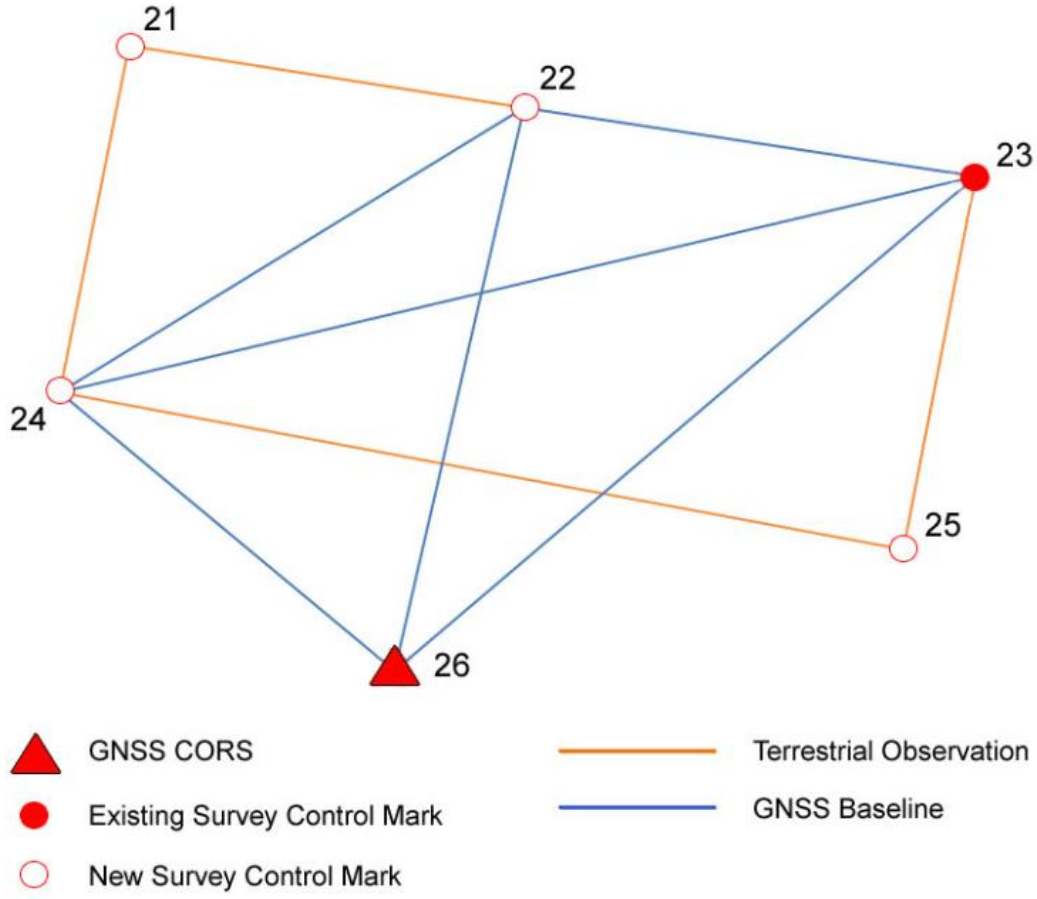

Figure 1: Combined GNSS and terrestrial control survey

All GNSS baseline measurements and uncertainties are based upon a two-hour observation period and have been extracted directly from baseline processing software. Each horizontal angle has been reduced from a set of three angle measurements.

Table 1: Station information

Station	Latitude	Longitude	AHD Height	Remarks
21	-35 58 47.8625	142 54 36.5997	103.70	New mark
22	-35 58 49.2624	142 54 48.7240	104.20	New mark
23	-35 58 51.1156	142 55 04.9316	104.10	AGRS mark
24	-35 58 59.3020	142 54 34.6274	103.60	New mark
25	-35 59 03.0482	142 55 02.8142	102.80	New mark
26	-35 59 07.5727	142 54 46.2340	112.897	AGRS GNSS CORS site

Table 2: Terrestrial measurement information

Measurement	Mark 1	Mark 2	Mark 3	Value	Std. dev.	Inst. ht.	Targ. ht.
Level diff. 1	21	22		0.506	0.010 m		
Level diff. 2	22	23		-0.045	0.011 m		
Level diff. 3	23	25		-1.320	0.011 m		
Level diff. 4	24	21		0.112	0.011 m		
Level diff. 5	25	24		0.754	0.015 m		
Level diff. 6	22	21		-0.506	0.010 m		
Level diff. 7	23	22		0.042	0.011 m		
Level diff. 8	25	23		1.318	0.011 m		
Level diff. 9	21	24		-0.112	0.011 m		
Level diff. 10	24	25		-0.740	0.015 m		
Slope dist. 1	21	22		306.790	0.010 m	1.650	1.651
Slope dist. 2	22	23		410.031	0.010 m	1.650	1.651
Slope dist. 3	24	21		356.003	0.010 m	1.650	1.651
Slope dist. 4	25	23		371.568	0.010 m	1.650	1.651
Slope dist. 5	25	24		715.488	0.010 m	1.650	1.651
Vt. angle 1	21	22		0 05 35.651	2.0”	1.650	1.651
Vt. angle 2	22	23		-0 00 25.657	2.0”	1.650	1.651
Vt. angle 3	23	25		-0 12 20.092	2.0”	1.650	1.651
Vt. angle 4	24	21		0 01 09.908	2.0”	1.650	1.651
Vt. angle 5	25	24		0 03 20.601	2.0”	1.650	1.651
Hz. angle 1	24	21	25	91 18 43.522	1.0”		
Hz. angle 2	25	24	23	88 55 20.712	1.0”		
Hz. angle 3	23	25	22	89 47 58.951	1.0”		
Hz. angle 4	22	23	21	180 04 31.199	1.0”		
Hz. angle 5	21	22	24	89 53 25.614	1.0”		

Table 3: GNSS measurement information

Measurement	Mark 1	Mark 2	Value	Precision (lower VCV)
GNSS 1	26	23	X -514.6419 Y -198.1707 Z 415.4426	7.843397e-07 -6.374967e-07 1.135554e-06 5.704006e-07 -7.829256e-07 1.083241e-06
GNSS 2	26	22	X -296.5955 Y 145.9824 Z 461.6305	4.395123e-07 -2.398766e-07 3.260701e-07 2.487716e-07 -2.461248e-07 4.525997e-07
GNSS 3	26	24	X 61.7913 Y 317.7330 Z 211.6262	3.443994e-07 -1.918564e-07 2.675435e-07 2.078026e-07 -2.246877e-07 4.216571e-07
GNSS 4	23	22	X 218.0438 Y 344.1575 Z 46.1858	1.078977e-06 -7.029027e-07 9.465813e-07 7.132949e-07 -7.109973e-07 1.097470e-06
GNSS 5	22	24	X 358.3865 Y 171.7508 Z -250.0044	6.455877e-07 -3.771552e-07 4.988322e-07 4.081710e-07 -3.921777e-07 6.773083e-07
GNSS 6	23	24	X 576.4315 Y 515.9078 Z -203.8178	1.415800e-06 -9.109323e-07 1.185005e-06 9.537250e-07 -9.403620e-07 1.468584e-06

To cater for the influence of gravity on the angle and spirit levelling measurements, ellipsoid-geoid separations (N) and deflections of the vertical in the prime meridian (ξ) and prime vertical (η) have been interpolated from AUSGeoid2020. The respective AUSGeoid2020 values for all marks are shown in Table 4.

Table 4: AUSGeoid2020 interpolated values

Station	N	ξ	η
21	4.512	-2.934	-2.535
22	4.515	-2.950	-2.541
23	4.518	-2.974	-2.576
24	4.507	-2.931	-2.513
25	4.512	-2.963	-2.544
26	4.506	-2.943	-2.508

6.1.1 Minimally constrained adjustment

In order to verify the GNSS measurement precisions, a minimally constrained adjustment incorporating only the GNSS measurements is executed. Holding survey control mark 22 fixed, the number of measurements is 18 (three per baseline) and the number of unknowns is 9 (three each for marks 23, 24, 26). This results in a DoF of 9. The minimally constrained adjustment at the 95% confidence level yields the results in Table 5.

Table 5: Minimally constrained adjustment results

Unknown parameters	9
Measurements	18
Degrees of freedom	9
Sigma zero	1.380
Upper limit	2.114
Lower limit	0.300

Using the global test, the network adjustment is deemed a success since the sigma-zero value (1.380) is less than the upper limit (2.114). Due to the high precision of the GNSS baseline measurements, the SU for all coordinates is less than 1 millimetre.

Using the local test, each baseline is examined to assess whether the correction exceeds the 95% critical value. A procedure for this test is to examine whether the normalised residual (calculated by dividing the measurement correction by the standard deviation of the correction) exceeds the critical value of the unit Normal distribution at 95%, which is 1.96. From this test, it is discovered that the X and Y components of baseline 1 fail (Table 6) since the normalised residuals exceed 1.96.

Table 6: Baseline components

Baseline	Part	Correction	Correction std. dev.	Normalised residual
1	ΔX	0.00131	0.00063	2.08 (fail)
1	ΔY	-0.00275	0.00085	-3.24 (fail)

In this example, it is assumed that all GNSS observations were taken under the same set of conditions and that these failures are most likely the consequence of over-optimistic measurement precisions, rather than baseline 1 being a gross error. In this case, it is appropriate to modify (i.e. scale) the *a-priori* uncertainties.

As a first attempt to rectify these failures, the sigma zero value (1.380) is used to scale all GNSS measurement uncertainties. This is based on the assumption that all GNSS baselines were derived under the same conditions and there are no gross errors. Following scaling of all measurements, the minimally constrained adjustment produces a new sigma zero value of unity. Repeating the adjustment yields a pass in the global test. However, the Y component of baseline 1 still fails the local test.

On close examination, it is identified that baseline 1 is receiving a larger than expected correction in the vertical direction. Upon scaling this baseline's variance matrix by partial matrix scalars of 1.0, 1.0 and 3.0, corresponding to the east-west, north-south and up axes, the X and Y components now pass the local test and the adjustment yields a sigma zero of 0.691. As the sigma zero value is significantly less than unity, indicating that the system of measurements is more precise than indicated by the supplied uncertainties, it is clear that the problem lies with the estimated precisions of baseline 1.

From this analysis, a better way to deal with this problem would be to rescale the variance matrix for baseline 1 by partial matrix scalars of 1.0, 1.0 and 5.0, and to rescale all other

baseline variance matrices back to their original values. Re-running the minimally constrained adjustment leads to all GNSS measurements passing the local test, and a sigma zero of 1.139 – which is a much more stable outcome and confirms that the precision for baseline 1 was indeed over optimistic.

95% SU and circular radius values for all marks arising from the minimally constrained GNSS-only adjustment are shown in Table 7.

Table 7: 95% SU values (minimally constrained GNSS-only adjustment)

Station	SU (E)	SU (N)	SU (Up)	Circular radius
22 (constrained)				
23	0.001	0.001	0.002	0.001
24	0.001	0.001	0.002	0.001
26	0.001	0.001	0.002	0.001

Having confirmed all GNSS measurements pass the local test, the remaining terrestrial measurements are introduced in order to verify the set of network measurements as a whole. The ellipsoid-geoid separations (N) and deflections of the vertical in the prime meridian (ξ) and prime vertical (η) shown in Table 4 are also introduced.

Again, a minimally constrained adjustment is executed holding survey control mark 22 fixed. In this case, the number of measurements is 43 and number of unknowns is 15, which includes the unknown coordinates of marks 21 and 25. This results in a DoF of 28. The minimally constrained adjustment at the 95% confidence level yields the following results in Table 8.

Table 8: Minimally constrained combined adjustment results

Unknown parameters	15
Measurements	43
Degrees of freedom	28
Sigma zero	0.778
Upper limit	1.588
Lower limit	0.547

This adjustment yields a pass in the global test and a pass in the local test for all GNSS and terrestrial measurements. As indicated by the lower than expected sigma zero value, many of the terrestrial measurements were better than anticipated. Whilst much time could be spent modifying the *a-priori* standard deviations to achieve a more stable result, the outcome is within the lower and upper limits and thereby deemed a success.

95% SU and circular radius values for all marks arising from the minimally constrained combined adjustment are shown in Table 9.

Table 9: 95% SU values (minimally constrained combined adjustment)

Station	SU (E)	SU (N)	SU (Up)	Circular radius
21	0.004	0.002	0.004	0.004
22 (constrained)				
23	0.001	0.001	0.002	0.001
24	0.001	0.001	0.002	0.001
25	0.004	0.007	0.006	0.007
26	0.001	0.001	0.002	0.001

If, for example, project specifications require all marks to have SU values no greater than 15 mm, the values in Table 9 indicate that the survey successfully achieved this objective.

6.1.2 Constrained adjustment to propagate datum and uncertainty

Having confirmed all measurements pass the local test, constraints are imposed on the adjustment to propagate datum and uncertainty from the AGRS. The adopted AGRS constraints are survey control mark 23 and GNSS CORS site (mark 26). The respective GDA2020 coordinates, AHD height and uncertainties for these marks are shown in Table 10. All uncertainties are at the 1 sigma (68.3% confidence level).

The published GDA2020 coordinates and standard deviations for survey control mark 23 have been estimated from a jurisdictional (state-wide) adjustment. The standard deviations in seconds translate to approximately 0.025 m in the north-south and east-west axes. Since a rigorously derived ellipsoid height value for mark 23 is unavailable, the AHD height is introduced as an approximation. A 0.2 m standard deviation has been adopted for AHD height for mark 23.

Table 10: AGRS constraints (1 sigma)

Constrain	Mar	Value	Std. dev.	Precision (lower VCV)
GDA2020	26	-4121849.2711 3115877.9599 -3726953.1897		1.526881e-06 -5.463696e-07 1.407678e-06 3.250046e-07 -1.977427e-07 1.250715e-06
GDA2020	23	-35 58 51.1179 142 55 04.9337	0.0008” 0.0008”	
AHD	23	104.1000	0.200m	

The GDA2020 (Cartesian) coordinates and uncertainties for the GNSS CORS site have been derived from routine analysis of Australia’s GNSS CORS sites. Hence, a rigorous, fully populated variance matrix is available. As shown by the lower VCV values in Table 10, the standard deviations are in the order of 1 mm. Since this is a rather optimistic estimate, in this example the VCV has been scaled by 7.5 to yield uncertainties at the 3 mm level.

For the constrained adjustment, all marks are held free and the constraints are introduced as measurements with their assumed uncertainties. The introduction of these constraints as measurements translates to a total measurement count of 49. Setting mark 22 as free

leads to 18 unknowns, resulting in a DoF of 31. The constrained adjustment at the 95% confidence level yields the results in Table 11.

Table 11: Constrained adjustment results

Unknown parameters	18
Measurements	49
Degrees of freedom	31
Sigma zero	0.735
Upper limit	1.556
Lower limit	0.566

This adjustment yields a pass in the global test and a pass in the local test for all measurements and constraints. As inferred from the lower than expected sigma zero value, the system of measurements were better than indicated by the prescribed uncertainties, and none of the constraints was shown to bias the adjustment in a significant way or to cause any of the measurements to fail.

95% PU and circular radius values for all marks arising from the constrained adjustment are shown in Table 12. From the foregoing analysis, the adjustment has proven to be a success.

Table 12: 95% PU values (constrained adjustment)

Station	PU (E)	PU (N)	PU (Up)	Circular radius
21	0.006	0.006	0.009	0.008
22	0.005	0.006	0.008	0.007
23	0.005	0.006	0.008	0.007
24	0.005	0.006	0.008	0.007
25	0.006	0.009	0.010	0.010
26	0.005	0.006	0.008	0.007

The preceding example has demonstrated how to evaluate coordinate and uncertainty estimates for a typical General Purpose Control Survey. In order to achieve the most rigorous estimates of position and uncertainty for survey control marks 21 – 25, the system of survey measurements (excluding the constraints) should be included within the jurisdiction adjustment.

The benefits of including the survey measurements within the jurisdiction adjustment cannot be over emphasised. For instance, consider AGRS survey control mark 23. Prior to this adjustment, the best published GDA2020 coordinates for mark 23 were stated as being precise to 0.025 m, 68.3% confidence level (or 0.049 m, 95%). However, according to Table 12 the estimated uncertainty for mark 23 based on the new system of measurements is 0.003 m, 68.3% (or 0.007 m 95%). Unless this survey is integrated within the AGRS, subsequent users of mark 23 will not be able to benefit from this increased precision.